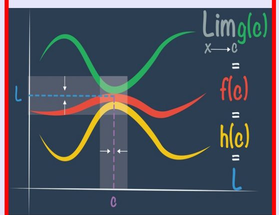
Calculus I

Lecture 42



Feb 19-8:47 AM

Suppose we want to find $\sqrt[5]{3}$

By Calc., $\sqrt[5]{3} \approx 1.246$

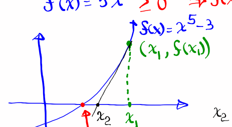
Let $x = \sqrt[5]{3}$, Raise both sides to the 5th power,

$x^5 = 3$, $\Rightarrow x^5 - 3 = 0$

Let $f(x) = x^5 - 3$

It is polynomial, diff. & Cont. $(-\infty, \infty)$

$f'(x) = 5x^4 \geq 0 \Rightarrow f(x)$ is increasing.



Eqn of tan. line

$y - y_1 = m(x - x_1)$

$y - f(x_1) = f'(x_1)(x - x_1)$

x_2 is x-Int. $\rightarrow y = 0$

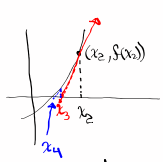
x-Int $\rightarrow y = 0 \rightarrow f(x) = 0 \rightarrow x^5 - 3 = 0$

$x = \sqrt[5]{3}$

$0 - f(x_1) = f'(x_1)(x - x_1)$

Divide by $f'(x_1) \neq 0$ $\qquad \dfrac{-f(x_1)}{f'(x_1)} = x - x_1$

Solve for x $\qquad x = x_1 - \dfrac{f(x_1)}{f'(x_1)}$ $\qquad x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$



By similar work

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$ $\qquad f'(x_2) \neq 0$

$x_4 = x_3 - \dfrac{f(x_3)}{f'(x_3)}$ $\qquad f'(x_3) \neq 0$

$\vdots$

I did asked you to google Newton's Method.

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\ ;\ f'(x_n) \neq 0}$$

Apr 30-8:46 AM

Back to our example, Find $\sqrt[5]{3}$

Let $x = \sqrt[5]{3} \rightarrow x^5 = 3 \rightarrow x^5 - 3 = 0$

using Calculator

$\sqrt[5]{3} \approx \boxed{1.245}73094\ldots \rightarrow 1.246$

Let $f(x) = x^5 - 3$

$f'(x) = 5x^4$

Newton's eqn $\rightarrow x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$$x_{n+1} = x_n - \frac{x_n^5 - 3}{5x_n^4} = \frac{5x_n^5 - (x_n^5 - 3)}{5x_n^4}$$

$$x_{n+1} = \frac{4x_n^5 + 3}{5x_n^4}$$

If $x_1 = 1$

$$x_2 = \frac{4x_1^5 + 3}{5x_1^4} = \frac{4(1)^5 + 3}{5(1)^4} = \frac{7}{5} = 1.4$$

$$x_3 = \frac{4x_2^5 + 3}{5x_2^4} = \frac{4(1.4)^5 + 3}{5(1.4)^4} \approx 1.276$$

$$x_4 = \frac{4x_3^5 + 3}{5x_3^4} = \frac{4(1.276)^5 + 3}{5(1.276)^4} \approx 1.247$$

$$x_5 = \frac{4x_4^5 + 3}{5x_4^4} = \frac{4(1.247)^5 + 3}{5(1.247)^4} \approx \boxed{1.246}$$

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Use Newton's method to Solve $x^5 - x - 1 = 0$

with $x_1 = 1$.

first guess

$f(x) = x^5 - x - 1$

$f'(x) = 5x^4 - 1$

Newton's eqn $\quad x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$$x_n = x_n - \frac{x_n^5 - x_n - 1}{5x_n^4 - 1} = \frac{x_n(5x_n^4 - 1) - (x_n^5 - x_n - 1)}{5x_n^4 - 1}$$

$$x_{n+1} = \frac{4x_n^5 + 1}{5x_n^4 - 1}$$

$x_1 = 1$

$$x_2 = \frac{4(1)^5 + 1}{5(1)^4 - 1} = \frac{5}{4} = 1.25$$

$$x_3 = \frac{4(1.25)^5 + 1}{5(1.25)^4 - 1} \approx 1.178$$

$$x_4 = \frac{4(1.178)^5 + 1}{5(1.178)^4 - 1} \approx 1.168$$

$$x_5 = \frac{4(1.168)^5 + 1}{5(1.168)^4 - 1} \approx 1.167$$

$$x_6 = \frac{4(1.167)^5 + 1}{5(1.167)^4 - 1} \approx 1.167$$

Solution to $x^5 - x - 1 = 0$ is ≈ 1.167

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Use Calculus to graph $f(x)=\dfrac{x}{x^2-1}$

1) Domain: All reals except ± 1

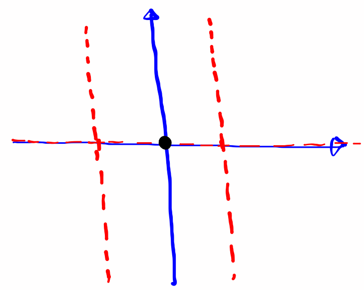
2) V.A. at $x=\pm 1$

3) Y-Int & x-Int at $(0,0)$

4) $f(-x)=\dfrac{-x}{(-x)^2-1}=\dfrac{-x}{x^2-1}=-f(x)$

$\Rightarrow$ odd function $\Rightarrow$ Symmetric w/t origin.

5) $\lim\limits_{x\to\pm\infty} f(x)=0 \Rightarrow$ H.A. at $y=0$



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$f(x)=\dfrac{x}{x^2-1}$

$f'(x)=\dfrac{1(x^2-1)-x\cdot 2x}{(x^2-1)^2}=\dfrac{-x^2-1}{(x^2-1)^2}=\dfrac{-(x^2+1)}{(x^2-1)^2}<0$

$f(x)$ is decreasing

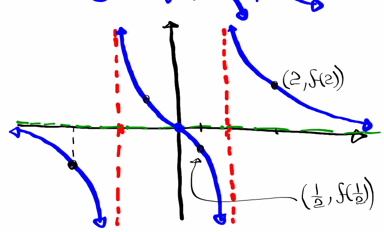
$f''(x)=\dfrac{-2x(x^2-1)^{\cancel{2}}--(x^2+1)\cdot 2\cancel{(x^2-1)}\cdot 2x}{(x^2-1)^{\cancel{4}}}$

$=\dfrac{-2x(x^2-1)+4x(x^2+1)}{(x^2-1)^3}=\dfrac{2x^3+6x}{(x^2-1)^3}$

$=\dfrac{2x(x^2+3)}{(x^2-1)^3}$ $\qquad f''(x)=0 \to 2x=0 \to x=0$

P.I.P. at $x=0$

x	$-\infty$		-1		0		1		∞
$f'(x)$		$-$	0	$-$		$-$	0	$-$	
$f''(x)$		$-$	0	$+$	•	$-$	0	$+$	
$f(x)$		↘		↘		↘		↘	



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Find a point on the graph of $xy=8$ that is closest to $(3,0)$

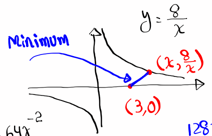
Hyperbola

$y=\frac{8}{x}$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(x-3)^2+\left(\frac{8}{x}-0\right)^2}$

$=\sqrt{(x-3)^2+\frac{64}{x^2}}$



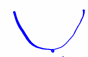
$64x^{-2}$

Let $f(x)=(x-3)^2+\frac{64}{x^2}$

$128x^{-3}$

$f'(x)=2(x-3)^1\cdot 1-\frac{128}{x^3}$

$f''(x)=2+128\cdot 3x^{-4}=2+\frac{384}{x^4}>0 \rightarrow$ C.U.



min.

$f'(x)=0$

$f'(x)=0 \quad 2(x-3)-\frac{128}{x^3}=0$

$2x^3(x-3)-128=0$

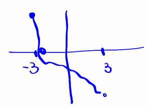
$x^4-3x^3-64=0$

Has 4 Solutions

Is $x=2$ a Solution? $\quad 2^4-3(2)^3-64=$
$16-24-64\neq 0$ NO.

Is $x=3$ a Solution $\quad 3^4-3\cdot 3^3-64=$
$81-81-64\neq 0$ No

what about $x=-3$ $\quad (-3)^4-3(-3)^3-64=$
$81+81-64\neq 0$ NO



use Newton's Method with $x_1=-2$ to find a Solution.

$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$

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